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COMPLEMENT AND μ-COMPLEMENT OF A REGULAR FUZZY GRAPH

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ABSTRACT

In this paper, various fuzzy graphs with regular and totally regular are introduced. Some theorems of regular fuzzy graphs are discussed with their complements and μ-complements. A necessary and sufficient condition under which they are equivalent is provided.

KEYWORDS: Strong Fuzzy Graph, Complete Fuzzy Graph, Regular Fuzzy Graph, Totally Regular Fuzzy Graph, Complement of a Fuzzy Graph and μ-Complement of a Fuzzy Graph

INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Though it is very young, it has been growing fast and has numerous applications in various fields. During the same time Yeh and bang have also introduced various connectedness concepts in fuzzy graphs. Moderson (1994) introduced the concept of complement of fuzzy graphs. M. S. Sunitha and A. Vijayakumar (2002) gave a modified definition of complement of fuzzy graph. A. Nagoorgani and V. T. Chandrasekaran (2009), defined μ–complement of a fuzzy graph, which is slightly different from the definition of complement of a fuzzy graph discussed by M. S. Sunitha and A. Vijayakumar. A. Nagoorgani and K. Radha (2008) introduced the concept of regular fuzzy graph. In this paper introduce the strong and complete fuzzy graphs with regular and also totally regular. We provide some theorems of regular fuzzy graphs with their complements and μ-complements through various examples.

First we go through some basic definitions in the next section.

BASIC CONCEPTS

Fuzzy Graph [1]

A fuzzy subset of a set V is a mapping σ from V to [0, 1]. A fuzzy graph G is a pair of functions G: (σ, μ) where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all x, y ∈ V. The underlying crisp graph of G: (σ, μ) is denoted by G*: (V, E) where $E \subseteq V\times V$. We write μ(xy) for μ(x, y).

Degree of a Vertex [1]

Let G: (σ, μ) be a fuzzy graph on G^*: (V, E). The degree of a vertex u is $d_G(u) = \sum_{u\neq v} \mu(u,v)$. Since μ(u, v) > 0 for uv ∈ E, μ(u, v) = 0 for uv ∉ E. This is equivalent to $d_G(u) = \sum_{uv\in V} \mu(u,v)$. The minimum degree of G is $\delta(G) = \wedge\{d_G(v), \forall\, v \in V\}$ and the maximum degree of G is $\Delta(G) = \vee\{d_G(v), \forall\, v \in V\}$.

Total Degree of a Fuzzy Graph [1]

Let G: (σ, μ) be a fuzzy graph on G^*: (V, E). The total degree of a vertex u∈ V is defined by

$td_G(u) = \sum_{u \neq v} \mu(u, v) + \sigma(u)$. Since $\mu(u, v) > 0$ for $uv \in E$, $\mu(u, v) = 0$ for $uv \notin E$. This is equivalent to $td_G(u) = d_G(u) + \sigma(u)$.

Regular Fuzzy Graph [1]

Let G: (σ, μ) be a fuzzy graph on G^*: (V, E). If $d_G(v) = k$ for all $v \in V$, (i.e) if each vertex has same degree k, then G is said to be a regular fuzzy graph or k – regular fuzzy graph.

Totally Regular Fuzzy Graph [1]

Let G: (σ, μ) be a fuzzy graph on G^*: (V, E). The total degree of each vertex has same degree k, then G is said to be a totally regular fuzzy graph or k – totally regular fuzzy graph.

Order and Size of a Fuzzy Graph [1]

The order and size of a fuzzy graph G are defined by $O(G) = \sum_{u \in V} \sigma(u)$ and $S(G) = \sum_{uv \in E} \mu(uv)$.

Remark

Let G: (σ, μ) be a fuzzy graph. Then $\sum_{v \in V} d(v) = 2 \sum_{uv \in E} \mu(uv) = 2S(G)$.

Note [3]

The underlying crisp graph of the fuzzy graph G: (σ, μ) is denoted as G^*: (σ^*, μ^*), where $\sigma^* = \{u \in S / \sigma(u) > 0\}$, $\mu^* = \{uv \in V \times V \mid \mu(uv) > 0\}$.

Strong Fuzzy Graph [3]

A fuzzy Graph G is strong, if $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$, where xy denotes the edge and for all $xy \in \mu^*$.

Complete Fuzzy Graph [3]

A fuzzy Graph G is complete, if $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$, where xy denotes the edge and for all $x, y \in \sigma^*$.

Complement of a Fuzzy Graph [5]

Let G: (σ, μ) be a fuzzy graph. The complement of G is defined as $\overline{G} : (\sigma, \overline{\mu})$, where $\overline{\mu}(xy) = \sigma(x) \wedge \sigma(y) - \mu(xy)$, $x, y \in V$.

Theorem *

The complement of a strong fuzzy graph is also strong.

Proof

Let G: (σ, μ) be a strong fuzzy graph.

To prove: complement of G: (σ, μ) is also strong.

By the definition of complement,

$\overline{\mu}$ (xy) = σ(x) ∧ σ(y) - μ(xy), x, y ∈ V.

= σ(x) ∧ σ(y) - σ(x) ∧ σ(y), μ(xy) > 0

σ(x) ∧ σ(y), μ(xy) = 0.

= 0, μ(xy) > 0

σ(x) ∧ σ(y), μ(xy) = 0.

= 0, $\overline{\mu}$ (xy) = 0

σ(x) ∧ σ(y), $\overline{\mu}$ (xy) > 0.

$\overline{\mu}$ (xy) = σ(x) ∧ σ(y), $\overline{\mu}$ (xy) > 0, for all x, y ∈ V, Where xy denotes the edge and for all xy ∈ $\overline{\mu}^*$.

Thus, the complement of a strong fuzzy graph is also strong.

REGULAR AND TOTALLY REGULAR FUZZY GRAPHS WITH THEIR COMPLEMENTS

Regular Strong Fuzzy Graph

A fuzzy graph G: (σ, μ) is said to be regular strong fuzzy graph, if it is satisfies the following conditions,

- μ(xy) = σ(x) ∧ σ(y) for all x, y ∈ V, where xy denotes the edge and for all xy ∈ μ^*.
- Each vertex of G has the same degree.

Totally Regular Strong Fuzzy Graph

A fuzzy graph G: (σ, μ) is said to be totally regular strong fuzzy graph, if it is satisfies the following conditions,

- μ(xy) = σ(x) ∧ σ(y) for all x, y ∈ V. where xy denotes the edge and for all xy ∈ μ^*.
- Each vertex of G has the same total degree.

Regular Complete Fuzzy Graph

A fuzzy graph G:(σ, μ) is said to be regular complete fuzzy graph, if it is satisfies the following conditions,

- μ(xy) = σ(x) ∧ σ(y) for all x, y ∈ V, where xy denotes the edge and for all x, y ∈ σ^*.
- Each vertex of G has the same degree.

Totally Regular Complete Fuzzy Graph

A fuzzy graph G:(σ, μ) is said to be totally regular complete fuzzy graph, if it is satisfies the following conditions,

- μ(xy) = σ(x) ∧ σ(y) for all x, y ∈ V. where xy denotes the edge and for all x, y ∈ σ^*.
- Each vertex of G has the same total degree.

Theorem

Let μ(xy) = [σ(x) ∧ σ(y)]/2, for all x, y ∈ V, where xy denotes the edge and for all x, y ∈ σ^*. Then G: (σ, μ) be a k – regular fuzzy graph if and only if complement of G: (σ, μ) is also k – regular,

Proof

Given $\mu(xy) = [\sigma(x) \wedge \sigma(y)]/2$, for all x, y $\in$ V, where xy denotes the edge and for all x, y $\in \sigma^*$.

$\therefore$ Then G^* is complete.

Let G: (σ, μ) be a k – regular fuzzy graph.

Then d(x) = k, for all x $\in$ V.

To prove: $\overline{G} : (\sigma, \overline{\mu})$ is also k – regular.

(i.e) to prove that d(x) = k, for all x $\in$ V in $\overline{G}$

It is enough to prove that $\mu(xy) = \overline{\mu}(xy)$.

Since G is regular.

Then d(x) = k, for all x $\in$ V.

$\therefore$ $k = d(x) = \sum_{x \neq y} \mu(xy)$, for all y $\in$ V. (1)

Since $\mu(xy) = [\sigma(x) \wedge \sigma(y)]/2$, for all x, y $\in$ V. (2)

By the definition of complement,

$\overline{\mu}(xy) = \sigma(x) \wedge \sigma(y) - \mu(xy)$, x, y $\in$ V.

$\therefore$ $\overline{\mu}(xy) = [\sigma(x) \wedge \sigma(y)]/2$, [by (2)]

$= \mu(xy)$. (3)

$\therefore$ $d(x) = \sum_{x \neq y} \overline{\mu}(xy) = \sum_{x \neq y} \mu(xy) = k$, for all y $\in$ V.

Hence $\overline{G} : (\sigma, \overline{\mu})$ is also k – regular.

Conversely,

Assume that, $\overline{G} : (\sigma, \overline{\mu})$ is k – regular.

To prove G: (σ, μ) is also k – regular.

It is enough to prove that $\mu(xy) = \overline{\mu}(xy)$.

By (3), $\overline{\mu}(xy) = \mu(xy)$.

$\therefore$ $\mu(xy) = \overline{\mu}(xy)$.

Hence G: (σ, μ) is also k – regular.

Note

In general, the above theorem is not true.

Remark

The converse of above theorem is not true.

In the following figures, G: (σ, μ) and $\overline{G}$: (σ, $\overline{\mu}$) are regular , but μ(xy) ≠ [σ(x) ∧ σ(y)]/2, for all x, y ∈ V, where xy denotes the edge and for all x, y ∈ σ^*.

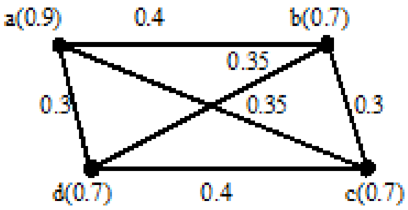

Figure 1: G: (σ, μ)

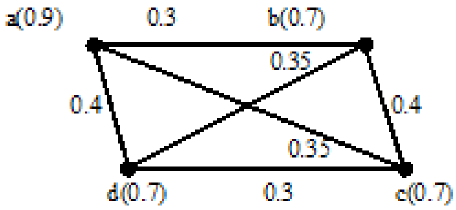

Figure 2: $\overline{G}$ **: (σ,** $\overline{\mu}$ **)**

Theorem

If G: (σ, μ) be a regular complete fuzzy graph, then the complement of G: (σ, μ) is also regular.

Proof

Given G: (σ, μ) be a regular complete fuzzy graph.

Then to prove that, $\overline{G}$: (σ, $\overline{\mu}$) is a regular.

By the definition of complement,

$\overline{\mu}$(xy) = σ(x) ∧ σ(y) - μ(xy), x, y ∈ V.

Since G: (σ, μ) be a regular complete fuzzy graph.

Then μ(xy) = σ(x) ∧ σ(y), for all x, y in σ^*.

∴ $\overline{\mu}$(xy) = σ(x) ∧ σ(y) - μ(xy), for all x, y ∈ V.

= σ(x) ∧ σ(y) - σ(x) ∧ σ(y), for all x, y ∈ V.

= 0, for all x, y ∈ V.

∴ d(x) = 0, for all x, y ∈ V.

Hence $\overline{G}$: (σ, $\overline{\mu}$) is a regular.

Remark

By the hypothesis of above theorem, $\overline{G}$: (σ, $\overline{\mu}$) is a regular, but not complete.

Remark

The converse of above theorem is not true.

In the following figures, $\overline{G}$: (σ, $\overline{\mu}$) is regular, but G: (σ, μ) is not complete regular.

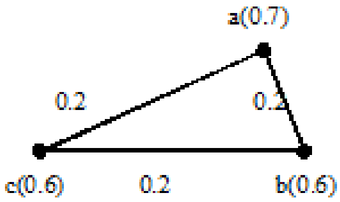

Figure 3: G: (σ, μ)

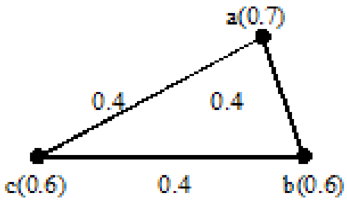

Figure 4: $\overline{G}$: (σ, $\overline{\mu}$)

RESULTS

If G^* is a regular graph, then complement of G^* is also regular.

Theorem

Let G: (σ, μ) be a regular strong fuzzy graph. Then $\overline{G}$: (σ, $\overline{\mu}$) is also a regular strong fuzzy graph, if σ is a constant function.

Proof

Let G: (σ, μ) be a regular strong fuzzy graph. (1)

Given σ is a constant function.

Then to prove, $\overline{G}$: (σ, $\overline{\mu}$) is a regular strong fuzzy graph.

If σ is a constant function, then μ is also a constant function. (2)

By (1) & (2), σ = μ = c.

Since G is a regular strong fuzzy graph.

Then d(x) = k, for all $x \in V$.

∴ d(x) = k = k_1c, for all $x \in V$.

∴ k_1 is a constant (since k is a constant).

∴ G^* is k_1 – regular. (3)

∴ The crisp graph of $\overline{G}$ is also regular, i.e., k_2 – regular. [by (1) & (3)] (4)

By the definition of complement, $\overline{\mu}(xy) = \sigma(x) \wedge \sigma(y) - \mu(xy)$, for all $x, y \in V$.

By (1) & (2),

$\overline{\mu}(xy) = c \wedge c - c$, for all $\mu(xy) \in \overline{\mu}^*$.

$c \wedge c$, otherwise.

∴ $\overline{\mu}(xy) = c$ for all $xy \in \overline{\mu}^*$.

∴ $\overline{G}$ is also strong fuzzy graph.

Thus d(x) = k_2c in $\overline{G}$, [by (4)].

∴ d(x) = m in $\overline{G}$, where m is a constant.

Hence $\overline{G}$ is also a regular strong fuzzy graph.

Remark

In the following figures, the converse part of above theorem is not true.

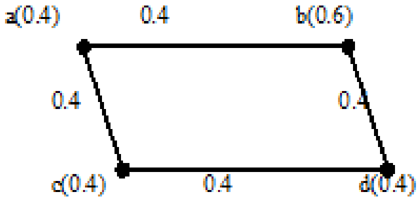

Figure 5: G: (σ, μ)

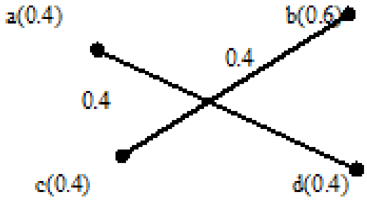

Figure 6: $\overline{G}$: (σ, $\overline{\mu}$)

From the above figures, G: (σ, μ) and $\overline{G}$: (σ, $\overline{\mu}$) are regular strong fuzzy graphs, but σ is not a constant function.

Theorem

Let G: (σ, μ) be a regular fuzzy graph. Then the following conditions are equivalent: (1) complement of G: (σ, μ) is regular, (2) complement of G: (σ, μ) is totally regular if and only if σ is a constant function.

Proof

Suppose that σ is a constant function.

Let σ(u) = c, a constant, for all u ∈ V.

Assume that $\overline{G}$: (σ, $\overline{\mu}$) is a k_1-regular fuzzy graph.

Then d(u) = k_1, for all u ∈ V.

So td(u) = d(u) + σ(u), for all u ∈ V.

∴ td(u) = k_1 + c, for all u ∈ V.

Hence $\overline{G}$: (σ, $\overline{\mu}$) is totally regular strong fuzzy graph.

Hence (1) ⇒ (2) proved.

Now, suppose that $\overline{G}$: (σ, $\overline{\mu}$) is a k_2-totally regular strong fuzzy graph.

Then td(u) = k_2, for all u ∈ V.

∴ d(u) + k_1 = k_2, for all u ∈ V.

∴ d(u) = k_2 - c, for all u ∈ V.

So $\overline{G}$: (σ, $\overline{\mu}$) is a regular fuzzy graph.

Hence (2) ⇒ (1) is proved.

Thus, (1) and (2) are equivalent.

Conversely, assume that (1) and (2) are equivalent.

(i.e)., $\overline{G} : (\sigma, \overline{\mu})$ is regular strong fuzzy graph if and only if $\overline{G} : (\sigma, \overline{\mu})$ is totally regular strong fuzzy graph.

Let $\overline{G}$ be a regular fuzzy graph. Then d(u) = a, for all u $\in$ V, where 'a' is a constant.

Since $\overline{G}$ is also totally regular strong fuzzy graph.

Then td(u) = d(u) + σ(u) = a + σ(u) = b , for all u $\in$ V.

$\Rightarrow$ a + σ(u) = b**,** for all u $\in$ V.

$\Rightarrow$ σ(u) = b - a, for all u $\in$ V, since b - a is also a constant.

Hence σ is a constant function.

Theorem

Let G: (σ, μ) be a strong fuzzy graph and σ be a constant function. Then G: (σ, μ) is both regular and totally regular if and only if complement of G: (σ, μ) is also both regular and totally regular.

Proof

Let G: (σ, μ) be a regular strong fuzzy graph and σ be a constant function. (1)

Then complement of G: (σ, μ) is also strong. (By theorem *) (I)

Assume that G: (σ, μ) is both regular and totally regular.

Then, to prove that complement of G: (σ, μ) is also both regular and totally regular.

By (1), we take σ(x) = c, for all x $\in$ V, where c is any constant. (2)

Since G: (σ, μ) be a regular strong fuzzy graph.

Then d(x) = $\sum_{x \neq y} \mu(xy)$ = kc, for all x, y $\in$ V.

Thus, G^* is regular.

Suppose, we take G^* is k – regular.

Let n be the number of vertices in G^*.

Then $\overline{G}^*$ is (n - k - 1) – regular. (3)

By the definition of complement,

$\overline{\mu}$(x, y) = $\sigma(x) \wedge \sigma(y) - \sigma(x) \wedge \sigma(y)$, μ(x, y) > 0.

= $\sigma(x) \wedge \sigma(y)$, μ(x, y) = 0.

(Since $\mu(x, y) = \sigma(x) \wedge \sigma(y)$)

$\overline{\mu}$(x, y) = 0, for all xy $\in$ μ^*.

= c, for all xy $\in$ $\overline{\mu}^*$.

$\therefore$ $d(x) = \sum_{x \neq y} \overline{\mu}(xy)$ = (n - k - 1)c, for all x, y $\in$ V.

Hence $\overline{G} : (\sigma, \overline{\mu})$ is regular.

Next to prove that $\overline{G} : (\sigma, \overline{\mu})$ is also totally regular.

Since σ is a constant function, then σ(x) = c, for all x $\in$ V, where c is any constant.

We take d(x) = k, for all x $\in$ V. (Since $\overline{G}$ is regular)

td(x) = d(x) + σ(x), for all x $\in$ V.

$\therefore$ td(x) = k + c, for all x $\in$ V.

Thus, td(x) = k_1, for all x $\in$ V, where k_1 is another any constant.

Hence $\overline{G} : (\sigma, \overline{\mu})$ is also totally regular.

In the same way and using (I), similarly to prove the converse part.

Hence the theorem.

REGULAR AND TOTALLY REGULAR FUZZY GRAPHS WITH THEIR μ-COMPLEMENTS

Theorem ** [3]

Let G: (σ, μ) be a fuzzy graph. Then G^{μ} is an isolated fuzzy graph iff G: (σ, μ) is a strong fuzzy graph.

Theorem ***

Let G: (σ, μ) be a complete fuzzy graph. Then μ-complement of G: (σ, μ) is isolated one.

Proof

Let G: (σ, μ) be a complete fuzzy graph.

To prove G^{μ}: (σ, μ^{μ}) is isolated fuzzy graph.

By the definition of μ-complement,

$\mu^{\mu}(xy) = \sigma(x) \wedge \sigma(y) - \mu(xy)$, if $\mu(xy) > 0$.

$\qquad\qquad 0 \qquad\qquad$, if $\mu(xy) = 0$.

Since G: (σ, μ) be a complete fuzzy graph.

Then, $\mu^{\mu}(xy) = 0$, for all x, y $\in$ V.

$\therefore$ G^{μ}: (σ, μ^{μ}) is an isolated fuzzy graph.

Remark

The converse of above theorem is not true.

Theorem

Let $\mu(xy) = [\sigma(x) \wedge \sigma(y)]/2$, for all x, y $\in$ V, where xy denotes the edge and for all xy $\in \mu^*$. Then G: (σ, μ) be a k – regular fuzzy graph if and only if G^μ: (σ, μ^μ) is also k – regular.

Proof

Given $\mu(xy) = [\sigma(x) \wedge \sigma(y)]/2$, for all x, y $\in$ V, where xy denotes the edge and for all xy $\in \mu^*$.

Let G: (σ, μ) be a k – regular fuzzy graph.

Then d(x) = k, for all x $\in$ V.

To prove: G^μ: (σ, μ^μ) is also k – regular.

(i.e) to prove that d(x) = k, for all x $\in$ V in G^μ.

It is enough to prove that $\mu^\mu(xy) = \mu(xy)$.

Since G is k – regular.

Then d(x) = k, for all x $\in$ V.

$\therefore$ k = d(x) = $\sum_{x \neq y} \mu(xy)$, for all y $\in$ V. (1)

Since $\mu(xy) = [\sigma(x) \wedge \sigma(y)]/2$, for all x, y $\in$ V, where xy denotes the edge and for all xy $\in \mu^*$. (2)

By the definition of μ-complement,

$\mu^\mu(xy) = \sigma(x) \wedge \sigma(y) - \mu(xy)$, if $\mu(xy) > 0$.

0 , if $\mu(xy) = 0$.

$\therefore$ $\mu^\mu(xy) = [\sigma(x) \wedge \sigma(y)]/2$, [by (2)]

$= \mu(xy)$. (3)

$\therefore$ d(x) = $\sum_{x \neq y} \mu^\mu(xy) = \sum_{x \neq y} \mu(xy)$ = k, for all x, y $\in$ V.

Hence G^μ: (σ, μ^μ) is also k – regular.

Conversely,

Assume that G^μ: (σ, μ^μ) is k – regular.

To prove: G: (σ, μ) is k – regular.

(i.e) to prove that d(x) = k, for all x $\in$ V in G.

It is enough to prove that $\mu(xy) = \mu^\mu(xy)$.

By (3), $\mu^\mu(xy) = \mu(xy)$. Therefore $\mu(xy) = \mu^\mu(xy)$.

Hence G^μ: (σ, μ^μ) is also k – regular.

Note

In general, the above theorem is not true.

Remark

The converse part of above theorem is not true. In the following figures, G: (σ, μ) and G^{μ}: (σ, μ^{μ}) are regular, but μ(xy) ≠ [σ(x) ∧ σ(y)]/2, for all x, y ∈ V, where xy denotes the edge and for all x, y ∈ μ^*.

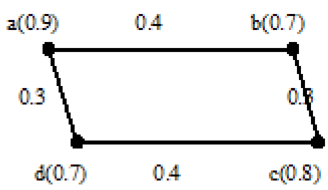

Figure 7: G: (σ, μ)

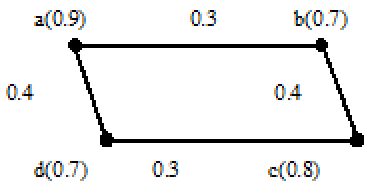

Figure 8: G^{μ}: (σ, μ^{μ})

Theorem

If G: (σ, μ) be a regular strong or complete fuzzy graph, then the μ-complement of G: (σ, μ) is also regular.

Proof

Using theorem ** and theorem ***, proof is trivial.

Remark

The converse of above theorem is not true.

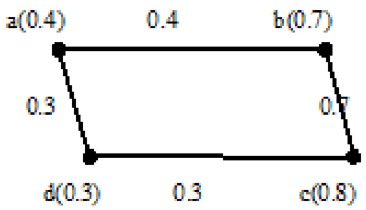

Figure 9: G: (σ, μ)

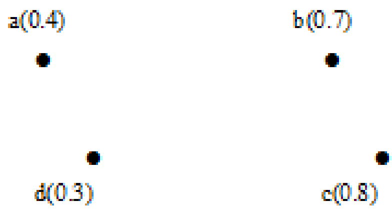

Figure 10: G^{μ}: (σ, μ^{μ})

From the above figures, G^{μ}: (σ, μ^{μ}) is isolated one and G: (σ, μ) is strong, but not regular. It is the same in the case of complete.

Theorem

Let G: (σ, μ) be a regular fuzzy graph. Then the following conditions are equivalent: (1) μ-complement of G: (σ, μ) is regular, (2) μ-complement of G: (σ, μ) is totally regular if and only if σ is a constant function.

Proof

Suppose that σ is a constant function.

Let σ(u) = c, a constant, for all u ∈ V.

Assume that G^{μ}: (σ, μ^{μ}) is a k_1-regular fuzzy graph.

Then d(u) = k_1, for all u ∈ V.

So td(u) = d(u) + σ(u), for all u ∈ V.

$\therefore$ td(u) = k_1 + c, for all u $\in$ V.

Hence G^μ: (σ, μ^μ) is totally regular strong fuzzy graph.

Hence (1) $\Rightarrow$ (2) proved.

Now, suppose that G^μ: (σ, μ^μ) is a k_2-totally regular strong fuzzy graph.

Then td(u) = k_2, for all u $\in$ V.

$\therefore$ d(u) + k_1 = k_2, for all u $\in$ V. Therefore d(u) = k_2 - c, for all u $\in$ V.

So G^μ: (σ, μ^μ) is a regular fuzzy graph.

Hence (2) $\Rightarrow$ (1) is proved.

Thus, (1) and (2) are equivalent.

Conversely, assume that (1) and (2) are equivalent.

(i.e) G^μ: (σ, μ^μ) is regular strong fuzzy graph if and only if G is totally regular strong fuzzy graph.

Let G^μ be a regular fuzzy graph.

Then d(u) = a, for all u $\in$ V, where 'a' is a constant.

Since G^μ is also totally regular strong fuzzy graph.

Then td(u) = d(u) + σ(u) = a + σ(u) = b , for all u $\in$ V.

$\Rightarrow$ a + σ(u) = b, for all u $\in$ V.

$\Rightarrow$ σ(u) = b - a, for all u $\in$ V, since b - a is also a constant.

Hence σ is a constant function.

PROPERTIES OF μ-COMPLEMENT OF REGULAR FUZZY GRAPHS

Theorem

If $\mu(xy) = [\sigma(x) \wedge \sigma(y)]/2$, for all x, y $\in$ V, where xy denotes the edge and for all xy $\in$ μ^*, then the size of any k – regular fuzzy graph and their μ-complement are same, that is, S(G) = S(G^μ) = nk/2, where n = $|V|$.

Proof

Given that $\mu(xy) = [\sigma(x) \wedge \sigma(y)]/2$, for all x, y $\in$ V, where xy denotes the edge and for all xy $\in$ μ^*. (1)

Then to prove that, S(G) = S(G^μ) = nk/2.

Since G: (σ, μ) is k – regular fuzzy graph. (2)

Then d(x) = k, for all x $\in$ V.

By the definition of size of a fuzzy graph, S(G) = $\sum_{xy\in E}\mu(xy)$.

WKT " $\sum_{x\in V} d(x) = 2\sum_{xy\in E}\mu(xy)$ ".

∴ $S(G) = (1/2) \sum_{x \in V} d(x) = (1/2)\, nk$, where n is the number of vertices in G^*.

∴ $S(G) = nk/2$.

Next to prove, $S(G^{\mu})$ is also equal to nk/2.

By (1) & (2), using the theorem 4.1, we have

$\mu^{\mu}(xy) = \mu(xy)$, for all $x, y \in V$, where xy denotes the edge and for all $xy \in \mu^*$.

∴ $G^{\mu}: (\sigma, \mu^{\mu})$ is also k – regular fuzzy graph.

Thus, $S(G^{\mu}) = nk/2$.

This completes the proof.

Remark

From the above theorem is true, in case of complement, if only possible $G: (\sigma, \mu)$ is also complete one.

Remark

The size of μ-complement of regular strong or complete fuzzy graph is zero.

Proof

Using theorem ** and theorem ***, proof is trivial.

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